ELECTRICAL MODELS FOR THE SOLUTION OF INVERSE NONSTEADY HEAT-CONDUCTION PROBLEMS

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Analog methods proposed earlier for the solution of inverse nonsteady heat-conduction problems on RC networks are further elaborated with allowance for convective, contact, and radiative heat transfer.

Interest has heightened lately in the subject of inverse heat-conduction problems, as evidenced by the publication of vast numbers of papers by Soviet and foreign authors on the solution of those problems. Along with the analytical methods discussed, for example, in [1, 2], mathematical modeling methods are used to great advantage for the solution of inverse problems. The applicability of electrical models for the solution of inverse problems becomes particularly evident in situations where, besides determining the boundary conditions from a known temperature, it is also required in the course of solution to estimate the influence of the accuracy with which the initial data are specified and of various kinds of nonlinearities in the domain of investigation and in the boundary conditions on the accuracy of the problem solution.

In [3] and in other papers by Kozdoba procedures based on the trial-and-error method are described for the solution of inverse nonsteady heat-conduction problems on R networks. The laborious manual selection of the boundary conditions for the solution of steady-state problems on R networks is eliminated by the use of an electromechanical servo system of the type discussed, for example, in [4].

At the present time RC networks are effectively used to obtain a time-continuous solution of direct nonsteady heat-conduction problems. As shown in [5-7], these models can be used to solve not only linear, but also nonlinear problems involving variable and nonlinear boundary conditions, even with regard for convective, contact, and radiative heat transfer. For the latter purpose the models must be complemented with special-purpose devices whose operation is based on the electronic simulation method. The use of this method in passive analog models makes it possible in generating the boundary conditions to simulate the various thermophysical coefficients involved in the boundary conditions by voltages. The author has also indicated [5] the possibility of using the electronic simulation method for the solution of inverse problems, and in [8] has discussed the design principles of devices for the solution on RC networks of linear and nonlinear inverse nonsteady heat-conduction problems subject to boundary conditions of the second and third kind when boundary conditions of the first kind are given. The use of RC networks for the solution of inverse nonsteady heat-conduction problems has made it possible, on the one hand, to enhance the efficiency of modeling methods and, on the other, to curtail the labor involved in the solution of the problems in question by those methods. For example, in the solution of inverse problems on RC networks it is no longer necessary to map the nonsteady temperature field over the entire modeling domain, as is unavoidable, for example, with the use of R networks.

Below we extend analog methods for the solution of inverse problems to the case of contact and radiative heat transfer. The initial data in this case are the temperatures of points, not only on the surface, but also interior to the investigated body.

In general, the solution of an inverse nonsteady heat-conduction problem is reduced to the determination of the conditions of external energy transfer between the body and the medium on the basis of the known temperatures $T_V(\tau)$ at certain interior points in the volume of the body. The external input of energy can be convective, conductive, or radiative. Consequently, depending on the type of boundary conditions, the

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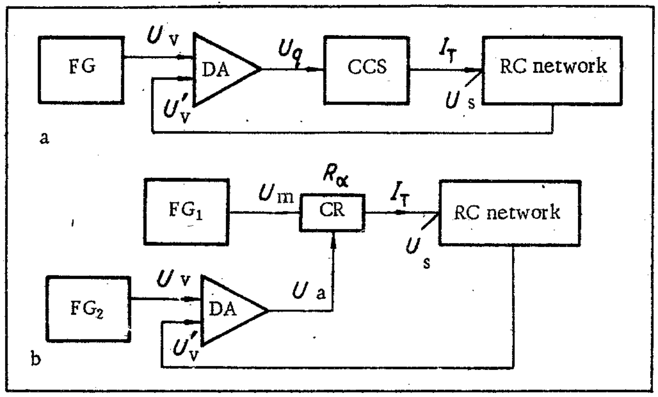

Fig. 1. Block diagram of the IPS device for the solution of inverse problems with variable boundary conditions of the second kind (a) and third kind (b).

solution of the inverse problem must yield the temperature $T_s(\tau)$ of the surface and the heat flux $q_s(\tau)$ across it, as well as the values of the heat-transfer coefficients α, thermal contact resistances R_c, and emissivities ε. Concurrently with the determination of the boundary conditions, as a rule, the temperature field of the entire body is reconstructed from measurements of the temperature at certain points thereof [1].

For the solution of inverse problems on analog computers, regardless of the method of solution chosen the model structurally represents a closed control system in which the controlled object is the network analog of the investigated body. The input parameters for the system in this case are the voltages $U_{Vi}(\tau_M)$ representing the electrical analogs of the temperatures $T_{V_i}(\tau)$ at certain points of the body, and the output parameters are the unknown currents, resistances, and voltages, i.e., the analogs of the heat fluxes and coefficients entering into the boundary conditions. If the trial-and-error method is used for solution [3], then the closed control loop of the given system includes an operator, who manually inspects the boundary conditions on the basis of an analysis of the modeling results until the voltages at the test points of the network model comply with the specifications. Of course, nonsteady problems can be solved by the given method only on R networks with quantization of the time variable.

For the solution of inverse problems on RC networks the system is assured of high performance by the inclusion in the closed control loop of high-speed automatic inverse-problem solvers (IPS), which, operating on a continuous-time basis without operator intervention, select the boundary conditions that will ensure the specified nonsteady temperature distribution in the body [8].

If it is required in the course of solution to determine only $q_s(\tau)$ and $T_s(\tau)$, the IPS network is fairly simple, including a differential amplifier (DA) with a high gain K_a and a controlled current stabilizer (CCS) (Fig. 1a). In this network the known voltage $U_V(\tau_M)$ simulating the specified temperature $T_V(\tau)$ is reproduced by the function generator (FG) and is delivered to one input of the DA. The second input of the DA is connected to an internal point of the RC network, where the voltage $U'_V(\tau_M)$ must vary according to the same law as $U_V(\tau_M)$. The DA output voltage $U_q = K_a(U_V - U'_V)$ is delivered to the input of the current stabilizer, where it is transformed into a proportional current $I_T = K_i U_q$. Inasmuch as the system is closed, for a large gain $K_a \to \infty$ the DA minimizes the voltage difference between U_V and U'_V in such a way as to reduce the system error $U = U_V - U'_V$ to zero. Since the voltage U'_V is determined solely by the current I_T flowing in the model from the CCS output, the equality of U_V and U'_V naturally indicates that the current I_T and so the voltage U_q are proportional to the unknown thermal flux q_s.

If the temperature $T_s(\tau)$ is used as the initial data, the solution of the problem is simplified, because now the flux $q_s(\tau)$ can be determined simply by measuring the current $I_T(\tau_M)$ flowing in the model directly from the FG output.

In situations where the heating of the body is induced by boundary conditions of the third kind, the knowledge of $q_s(\tau)$ is not sufficient for the specification of uniform boundary conditions at different points on the surface of the body. This is so because for equal values of the temperature $T_m(\tau)$ of the surrounding medium and of the heat-transfer coefficients $\alpha(\tau)$ the heat flux $q_s(\tau)$ is unequal at different points of the body because of the differences in $T_s(\tau)$. Consequently, for the solution of inverse problems with boundary conditions of the third kind it is advisable to determine $\alpha(\tau)$ along with $q_s(\tau)$. This can be done if the known

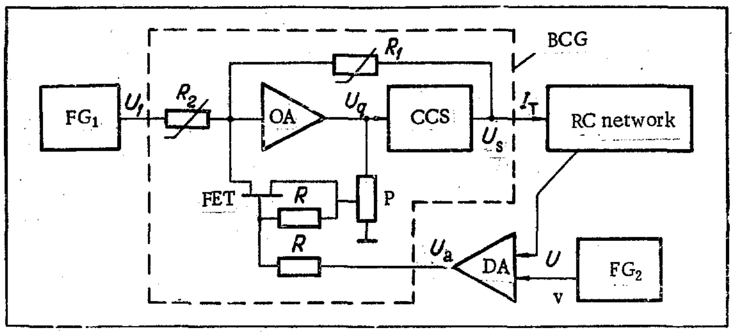

Fig. 2. Block diagram of a general-purpose device for the analysis of convective, contact, and radiative heat-transfer processes.

network specifying the boundary conditions of the third kind on the model includes as the boundary resistor R_α simulating α an inertialess controlled resistor CR, which receives its control signal from the DA (Fig. 1b). The CR can be any unipolar element capable of varying the internal resistance continuously and over a wide range as a function of the control voltage U_a. Because negative feedback is present in the IPS network, nonlinearity of the CR transformation characteristic does not affect the accuracy of the inverse problem solution. On the other hand, that nonlinearity renders more complex the measurement and principal reproduction of the required value of R_α at different points of the body. These problems can be eliminated by using linear CR for R_α, for example linearized field-effect transistors [9] or pulse-time controlled resistances [10].

The network discussed above can be used to solve inverse problems for both convective and contact heat transfer. In the latter case the FG_1 reproduces a voltage simulating the temperature $T_{c_1}(\tau)$ of the contact surface of one of the bodies as determined either experimentally or by reconstruction of the temperature field by the model solution of the inverse problem, while the FG_2 reproduces a voltage $U_V(\tau_M)$ proportional to the known temperature at a certain point of the second body. The model solution of the problem determines the time dependence of the quantity $R_c(\tau_M)$, which in the given case is the electrical analog of the nonsteady thermal contact resistance $R_c(\tau)$ of two plane surfaces. Knowing $R_c(\tau)$, one can then determine the dependence of R_c on the temperature of the contact zone.

As mentioned, in the solution of nonsteady heat-conduction problems with time-variable and nonlinear boundary conditions on RC networks it is advantageous to use devices designed on electronic simulation principles. Electronic simulation forms the basis of operation of the general-purpose device described below for the specification of boundary conditions ("boundary-condition generator," BCG), which can be used with RC networks to solve both direct and inverse problems for convective, contact, and radiative heat transfer. This device can also be used for the specification of nonlinear boundary conditions in cases where the nonlinear heat-conduction equation can be reduced by the Kirchhoff integral transformation to a form suitable for modeling on RC networks [6, 8].

The proposed BCG (Fig. 2) consists of an operational amplifier (OA) and controlled current stabilizer (CCS) connected in series and enclosed in a voltage feedback loop. The input signal to the device is the voltage $U_1(\tau_M)$ received from FG_1, and the output signal is the current $I_T(\tau_M)$ delivered to the end point of the RC network from the CCS output. The OA in conjunction with the input resistors R_1, R_2 and the feedback-connected linearized field-effect transistor FET [9] are designed to sum the input signals U_1 and U_s and to linearly multiply the resultant sum by a variable coefficient. In the solution of problems with nonlinear boundary conditions the elements R_1 and R_2 can be nonlinear or diode-controlled resistances with a volt-ampere characteristic determined by the conditions of the problem [10]. The variability of the coefficients involved in the boundary conditions can be accounted for by the appropriate time variation of the control voltage U_a delivered to the FET gate.

For the solution of inverse problems the BCG is augmented with a differential amplifier DA, which minimizes the difference between the voltages U_V and $U_V^{\prime}$ received at its inputs from the RC network and FG_2, respectively. The output voltage U_a of this amplifier is the solution of the inverse problem and, depending on the problem type, simulates one of the thermophysical variables: α, R_c or ε.

For the investigation of convective heat transfer

$$\alpha(\tau)\,[T_m(\tau) - T_s(\tau)] = q_s(\tau) \tag{1}$$

the given device makes it possible, in solving the inverse problem, to determine $\alpha(\tau)$ and $q_s(\tau)$ simultaneously. Here FG_1 is used to reproduce a voltage $U_m(\tau_M)$ proportional to the temperature $T_m(\tau)$ of the medium, and FG_2 reproduces a voltage $U_v(\tau_M)$ proportional to the given temperature $T_v(\tau)$.

Inasmuch as the IPS network includes a closed control loop, for a large value of $K_a \gg 1$ the voltage $U_v'(\tau_M)$ of an internal point of the model, being determined by the current $I_T(\tau_M)$, will be such that the difference between the voltages U_v and U_v' tends to zero at every instant. Of course, the current $I_T(\tau_M)$ and so also the voltage $U_q(\tau_M)$ are proportional in this case to the heat flux $q_s(\tau)$ in (1), i.e.,

$$U_q(\tau_M) = \frac{1}{K_i} I_T(\tau_M) \approx K_q K_\tau q_s(\tau). \tag{2}$$

On the other hand, when the condition $R_1 = R_2$ = const is satisfied, it is evident from the network that the indicated voltage is equal to

$$U_q(\tau) = K_{oa} \frac{1}{U_y}(\tau_M)\,[U_m(\tau_M) - U_s(\tau_M)], \tag{3}$$

where $K_{oa} = 1/R_1 K_s K_{ft}$ is the OA conversion coefficient, in volts.

If we transform from electrical to thermal quantities, we infer from the equality of Eqs. (2) and (3) that in convective heat transfer the voltage $U_a(\tau_M)$ simulates the heat-transfer coefficient $\alpha(\tau)$ in (1), i.e.,

$$U_y(\tau_M) = \frac{K_{oa} K_i}{K_q K_\tau} \cdot \frac{T_m(\tau) - T_s(\tau)}{q_s(\tau)} = K_\alpha \frac{1}{\alpha(\tau)}. \tag{4}$$

It is easily shown that in the investigation of contact heat transfer

$$\frac{1}{R_c}[T_{c_1}(\tau) - T_{c_2}(\tau)] = q_\kappa(\tau) \tag{5}$$

the device makes it possible, in solving the problem, to determine the physical quantities $R_c(T_c)$ and $q_c(\tau)$, whose electrical analogs in the model are, respectively, the voltages $U_a(\tau_M)$ and $U_q(\tau_M)$. Here FG_1 is used to model the surface temperature $T_{c_1}(\tau)$ of the first body, and FG_2 the known temperature $T_v(\tau)$ at some point in the volume of the second body.

In the solution of inverse problems for radiative heat transfer the emissivity ε can be determined at every instant from the Stefan—Boltzmann equation

$$\varepsilon(T)\,\sigma_0\,[T_m^4(\tau) - T_s^4(\tau)] = q_s(\tau). \tag{6}$$

For the modeling of the nonlinear equation (6) the network of the device incorporates as the elements R_1 and R_2 nonlinear resistances with volt-ampere characteristics of the form $I = aU^4$. They make it possible to eliminate special nonlinear transducers [6, 8] and to combine the operation of nonlinear transformation of the voltages U_1 and U_s with the summation operation. It is readily shown that the voltage $U_q(\tau_M)$ in this case is equal to

$$U_q(\tau_M) = K_{oa}' \frac{U_m^4(\tau_M) - U_s^4(\tau_M)}{U_a(\tau_M)}, \tag{7}$$

where $K_{oa}' = a/K_p K_{ft}$.

We have shown earlier that for a large value of K_a the voltage $U_q(\tau_M)$ is proportional to the heat flux $q_s(\tau)$ (2). In the solution of inverse radiative heat-transfer problems, therefore, the voltage $U_a(\tau_M)$ simulates the emissivity ε in (6), because

$$U_a(\tau_M) = \frac{K_{oa}' K_i^4}{K_q K_\tau} \cdot \frac{T_m^4(\tau) - T_s^4(\tau)}{q_s(\tau)} = K_\varepsilon \frac{1}{\varepsilon}(\tau). \tag{8}$$

The devices discussed above are made up of typical analog elements and can therefore be implemented on existing models, such as the general-purpose simulator USM-1, by a slight readjustment of their structure. The USM-1 comes with channels for boundary conditions of the first and second kind, as well as function generators [11], and these components can function as the amplifiers and current stabilizers in this case.

The accuracy of the solution of inverse problems depends in large measure on the accuracy with which the initial data are specified, the levels of the criteria Bi and Fo, and the distance of the internal points at which the temperature is known, from the surface of the body [1]. The latter condition is dictated by the fact that objects with spatially distributed parameters are characterized by finite response and a delay time that depends on the distance of the internal points from the surface of the body [12]. The presence of delay produces a time shift on the part of processes on the surface and inside the body and, as a result, instability on the part of direct methods for the solution of the investigated problems [13].

For the solution of inverse problems on electrical models by means of the devices described above the accuracy of the solution is determined the round-trip gain of the closed feedback loop. Increasing this gain, on the one hand, decreases the error of the solution and, on the other, induces excessive fluctuations in the system or a general loss of stability, even when boundary conditions of the first kind are specified. But if the temperatures of internal points of the body are introduced as the initial data, the presence of a delay in this case can diminish the model reproduction of the boundary conditions. Special measures must be adopted, therefore, to correct the system so as to increase the accuracy of the final solution and to improve its quality.

As the foregoing discussion implies, considerable importance attaches to the setting up of special investigations to determine the influence of the delay time, error in the specification of the initial data, the values of Bi and Fo, and various kinds of nonlinearities on the accuracy of the electrical model solution of inverse problems. The results of such investigations are of independent interest and are not discussed in the present article. It is important to note that from the standpoint of automatic control theory inverse heat-conduction problems are similar in their formulation to identification problems for objects with distributed parameters. The models used to solve these problems with a closed-loop network analog of the object are equivalent in their properties to automatic control systems. It is logical, therefore, to draw on the practicality of the methods of automatic control theory for the analysis of the results of the above-indicated investigations, as well as in the analysis of the model itself.

The solution of inverse problems by modeling methods is of interest not only from the thermophysical standpoint, but also from the standpoint of automatic control theory for the synthesis of an automatic controller that implements control of a distributed-parameter object according to a precalculated temperature at some point of that object. The specified law in this case can correspond to, for example, the optimum operational regime of the object. This law can be determined either by the mathematical tools of optimal control theory for distributed-parameter systems [14] or by mathematical modeling methods [15]. The use of a closed system for optimal control eliminates the obstructive influence of an unknown variable such as the heat-transfer rate and to offset various random disturbances.

We note in conclusion that when the geometry of the investigated body is more complex and, accordingly, the number of variables whose values must be determined by solution of the inverse problem increases, a number of factors can render automation of the process of solution on RC networks inefficient. Chief among these factors are the greater complexity of the solving algorithms and the concomitant complexity of the hardware, as well as the increase in the solving time. It is sometimes more practical, therefore, to replace problem-solving hardware operating to satisfy the fixed conditions of the problem with an operator, who intuitively decides the need for and the principle modes of modification of the executive program [16]. In this case, however, it is advisable to use RC networks for the solution of inverse nonsteady problems, augmenting them with appropriate devices for representation of the results of the solution with simultaneous appraisal of the quality of the solution, along with BCG devices, which make it possible even in the case of complex boundary conditions to execute the simple input and scanning of coefficients fed into them.

NOTATION

T is the temperature, °K;
q_s is the heat flux density at the surface of a body, W/m^2;
α is the heat-transfer coefficient, $W/m^2 \cdot {}^\circ K$;
σ_0 is the black-body emittance, equal to $5.67 \cdot 10^{-8}$ $W/m^2 ({}^\circ K)^4$;
ε is the black-body emissivity;
τ is the time, sec;
Fo is the Fourier number;
Bi is the Biot number;
U is the voltage, V;

I	is the current, A;
R	is the resistance, Ω;
K_i	is the CCS transfer constant, A/V;
K_q	is the conversion coefficient from q_s to U_q, m^2/A;
K_τ	is the conversion coefficient from τ to τ_M;
$K_\alpha = K_{oa}K_t/K_qK_\tau$	is the conversion coefficient from $1/\alpha$ to U_a, $V \cdot W/m^2 \cdot {}^\circ K$;
K_t	is the conversion coefficient from T to U, V/°K;
$K_\varepsilon = K'_{oa}K_t^4/K_qK_\tau\sigma_0$	is the conversion coefficient from $1/\varepsilon$ to U_a, V;
K_p	is the potentiometer (P) transfer constant;
K_{ft}	is the conversion coefficient of the linearized FET, $1/\Omega \cdot V$.

LITERATURE CITED

1. A. G. Temkin, Inverse Heat-Conduction Methods [in Russian], Énergiya, Moscow (1973).
2. O. M. Alifanov, Inzh.-Fiz. Zh., 24, No. 2 (1973).
3. L. A. Kozdoba, Electrical Modeling of Heat- and Mass-Transfer Effects [in Russian], Énergiya, Moscow (1972).
4. Yu. M. Matsevityi, in: Heat and Mass Transfer [in Russian], Vol. 8, ITMO Akad Nauk Belorussian SSR, Minsk (1972).
5. V. E. Prokof'ev, Inzh.-Fiz. Zh., 18, No. 1 (1970).
6. O. N. Suetin, O. T. Il'chenko, Yu. M. Matsevityi, and V. E. Prokof'ev, Izv. VUZ. Élektromekhan., No. 9 (1972).
7. V. E. Prokof'ev, Izv. VUZ. Élektromekhan., No. 4 (1973).
8. V. E. Prokof'ev, Inzh.-Fiz. Zh., 22, No. 2 (1972).
9. M. A. Rozenblat and A. M. Kreshtal, Avtomat. i Telemekhan., No. 4 (1971).
10. V. B. Smolov, Analog Computers [in Russian], Vysshaya Shkola, Moscow (1972).
11. N. S. Nikolaev, N. P. Polgorodnik, and E. S. Kozlov, The USM-1 Analog Mathematical Machine [in Russian], Mashgiz, Moscow (1962).
12. V. A. Makovskii, Dynamics of Metallurgical Objects with Distributed Parameters [in Russian], Metallurgiya, Moscow (1971).
13. J. V. Beck, Internat. J. Heat and Mass Transfer, 13, No. 4 (1970).
14. A. G. Butkovskii, S. A. Malyi, and Yu. N. Andreev, Optimal Control of the Heating of a Metal [in Russian], Metallurgiya, Moscow (1972).
15. K. I. Bogatyrenko, O. T. Il'chenko, O. T. Il'chenko, V. E. Prokof'ev, and O. N. Suetin, Inzh.-Fiz. Zh., 22, No. 4 (1972).
16. I. M. Vitenberg, High-Speed Analog Computers [in Russian], Énergiya, Moscow (1970).